

Starting with the Young-Laplace equation, we have:

$$\Delta p = -\gamma \cdot \hat{n} \quad (1.1)$$

Recognizing $\hat{n} = \vec{\nabla} f$, for the surface defined by f simplifies the previous expression to:

$$\Delta p = -\gamma \nabla^2 f$$

Expanding the previous expression yields (assume symmetry around θ):

$$\begin{aligned} \Delta p &= -\gamma \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{\partial^2 f}{\partial z^2} \right] \\ &= -\gamma \left[\frac{1}{r} \left(r \frac{\partial^2 f}{\partial r^2} + \frac{\partial f}{\partial r} \right) + \frac{\partial^2 f}{\partial z^2} \right] \end{aligned}$$

Which can be rewritten as:

$$\Delta p = -\gamma \left(\frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} + \frac{\partial^2 f}{\partial z^2} \right) \quad (1.2)$$

At this point, I see the general form is comparable to the weak condition in the square droplet case: <http://www.comsol.com/community/exchange/121/>, which has:

$$-\mu \left[\text{test}(uTr) + \text{test}(u)/r + \text{test}(wTz) \right]$$

And I understand that I get there by integrating Eq. (1.2) by parts with the TEST function. For example, if I take the first term, what I get is:

$$\int_a^b \frac{\partial^2 f}{\partial r^2} T(r) dr = \text{Constants} + \int_a^b \frac{\partial f(r)}{\partial r} \frac{\partial^2 T(r)}{\partial r} = \text{More Constants} + \int_a^b f(r) \frac{\partial^2 T(r)}{\partial r}$$

