# Mathematical model and transient solution



Figure 1: Schematic view of the fracture-matrix system.

As schematically illustrated in Figure 1, the system under consideration is the same as that studied by Tang et al. (1981) and Liu et al. (2017) for contaminant transport in fractured crystalline rock. The fracture is represented by two smooth parallel plates with a constant aperture $2b$, embedded in a semi-infinite homogeneous porous rock. The water velocity $u$ in the fracture is constant and the tracer is injected at the inlet of the fracture with a concentration of $c\_{in}\left(t\right)$.

The contaminant transport in the fracture is governed by advection, hydrodynamic dispersion, surface retardation, while the movement in the matrix is affected by diffusion and sorption. Combined with this conceptualization and other simplifications and assumptions used in Tang et al. (1981), the transport processes in the fracture-matrix system can be described by two coupled, one dimensional equations of continuity that can be written, respectively, as,



 (1)

and



 (2)

along with the initial and boundary conditions for contaminant transport through the fracture,



 (3)



 (4)



 (5)

and in the porous matrix,



 (6)



 (7)



 (8)

where the subscripts $f$ and $p$ refer to the fracture and pore space of the rock matrix, respectively; $x$ and $z$ are the coordinates along and perpendicular to the fracture plane, respectively; $t$ is the time; $c$ denotes the concentration of the contaminant; $R$ indicates retardation coefficient; $D$ represents the longitudinal dispersion coefficient in the fracture or the pore diffusion coefficient in the homogeneous matrix;$ ε$ indicates the porosity.

To demonstrate the accuracy of the solution, the results of Eq. (21) with the use of Eqs. (A5) and (A6) in Appendix A for $g\left(t-R\_{f}τ,Gτ\right)$ in the case of a Heaviside step injection ($c\_{in}\left(t\right)=1$ mol/m3) are presented in Fig. 4 for a fracture at $x=$ 0.76 m. The other parameters used are: $2b=$ 120 m, $ε\_{p}=$ 0.35, $u=$ 0.75 m/d, $D\_{f}=$ 6.6×10-6 m2/s, $D\_{p}$ is varied in the range from 0.0 to 10-10 m2/s, and surface retardations are not considered. In Fig.4 are also shown the breakthrough curves obtained from the solution of Tang et al. [1981] and from a numerical approach that transforms Eq. (9) back to the time domain by De Hoog algorithm [De Hoog et al., 1982] for the same case.



Fig. 4. Breakthrough curves for a fracture at *x* = 0.76 m, for *Dp* = 0.0, $10^{-14}$, $10^{-13}$, $10^{-12}$, $10^{-11}$ and $10^{-10}$ m2/s (from top to down), respectively; the concentration is normalized by *c*0=1 mol/m3.