Akif Ibragimov and Paul Mario Koola

Wave Carpet Simulation Using Coupled Hydro-Elastic FEMLAB Model

Abstract **- Wave Carpet is a novel deep offshore wave-power floating system concept funded by the Office of Naval Research that will have low overall life cycle cost due to an integrated design and be rapidly re-deployable. This design ensures better steady power output from the randomly fluctuating input wave power using built in energy storage and an internal electric grid. The Wave Carpet can also act as a wave damper thereby sharing the cost of power generated. In this paper we numerically model the Wave Carpet motion using a coupled hydro-elastic time-domain solution. The major feature of the mathematical model is the presence of non classical time dependent boundary conditions. The model is implemented in FEMLAB using weak and dweak formulation and coupled variable concept. Corresponding initial value problem is resolved in two-dimensions with certain restrictions on the initial distribution of the potential function. Typical results of the geometrical and physical parameters on energy distribution along the carpet are presented.**

I. INTRODUCTION

Solar radiation on the earth estimated to be about 10^{16} Watts is continuous and inexhaustible. The oceans, which covers about 70% of the earth's surface acts as a natural reservoir for this energy source. Solar radiation on the earth's surface creates pressure gradients in the atmosphere, which give rise[i] to winds, which blow over the ocean water mass to create waves. In order to design wave energy devices it is essential to understand the nature of these water waves, and the operation of these devices at sea. In this paper we will discuss the main issues and problems facing wave energy absorption from a standpoint of deep-water rapidly redeployable steady power wave energy design.

We present Wave Carpet a novel deep offshore wave-power floating system concept funded by the Office of Naval Research that will have low overall life cycle cost due to an integrated design. Unlike earlier wave power designs one of the major requirements is that this device has to be rapidly re-deployable. This device should be easier to maintain and since the device operates in deep water away from the mainland electric grid we have to ensure steady power output in spite of the randomly fluctuating input wave power, using built in energy storage.

A. *Device Designs:*

Three main categories of phenomenon can be utilized in wave energy conversion systems [1],[4],[5],[6].

1. Surface wave height or slope variations
2. Subsurface water particle movement or variations in pressure and
3. Wave transformation when approaching natural or artificial shoals.

Most of the devices invented so far, including the most promising systems, seem to belong to the first category. These include various types of wave-actuated buoys, oscillating bodies or bodies in which the waves cause pneumatic pressure variations. The devices in the second group have to face an exponentially decaying wave energy level below the surface. The investigations under the third category are site specific and have been very rare.

Another important categorization is based on the dimensions of the device compared to the wave and its orientation with respect to the wave [2]. They are the Terminator, Attenuator and Point Absorber. A terminator is a device that is aligned parallel to the wave front and perpendicular to the principal wave direction, thus terminating the waves. An attenuator lies in the principal wave direction and attenuates the waves as it passes by. A point absorber is a small device in plan compared to the wavelength. These devices can diffract the waves thereby capturing energy from a width greater than its own dimensions.

From a functional perspective devices can be categorized as dynamic and passive. In dynamic systems, one element of the system is tuned into excitation by the waves and can be optimized to extract maximum energy through proper tuning of system characteristics. Most of the popular devices fall under this category and some examples are the Cockerell Raft, Salter's Duck, Bristol cylinder, floating pistons and platforms and devices with oscillating water column. In passive systems there is not much possibility of tuning and examples include the TAPCHAN system [3], which uses a ramp to run up waves into a reservoir. These types of systems are highly site specific.

While it will be impossible to describe all these systems here what will be more useful is to analyze some of the

[i] Texas Tech University, Mathematics and Statistics, Box 41042, Lubbock TX 79409, BSI One KBSI Place, 1408 East University Dr. College Station, TX 77840, akif.ibraguimov@ttu.edu , aibragimov@kbsi.com, pkoola@kbsi.com

drawbacks of these systems with a view to improve future designs. Some of the main problems encountered with sea trials of these devices are:

- Hinges and components exposed to the sea are highly susceptible to corrosion. Moving components under water are extremely difficult to maintain.
- Train of adjacent floating devices requires accurate positioning techniques especially in extreme weather.
- Power is the product of force and velocity. Wave power is associated with very large forces but relatively low velocities. Design of components need to keep this enormous forces in mind.
- Wave Power is irregular and varies at different time scales: instantaneously, daily, monthly or seasonal. Storage at different time scales is critical to a deep-water design far away from the mainland electric grid.

Due to the above mentioned problems and the Navy requirements of rapid deployability and redeployability from site to site under all weather and operational conditions we present Wave Carpet as a novel concept design that could meet these added requirements.

II. The Concept –"Wave Carpet"

Long ago sailors had a practice of letting go of their fuel into the ocean to calm stormy seas [7], [8]. The Wave Carpet will be the modern day equivalent of the old-fashioned wave damper. It could be thought off as a flexible rubber mat square kilometers in size. Like the oil that spreads out around the ship protecting it, the wave carpet will spread out in all directions and be extremely large in dimensions by today's offshore standards. It will be a cross breed between a terminator and an attenuator (a termenuator) and will be just the opposite of the point absorber. Its size will be comparably larger than the largest wavelengths encountered making it a very large floating structure (VLFS). Current technologies of similar proportions are the Mobile Offshore Bases (MOB) being built for the US Navy and the Japanese seaports. The sheer size of the structure will provide the reaction required to oppose waves, since different ends of the structure will be subjected to nearly non synchronized loads due to phase differences of the wave at different regions. This should help simplify position keeping and we anticipate doing away with moorings and being self sufficient with dynamic positioning systems with minimal power to take care of long term drift forces. An artist's impression of the wave carpet is shown Figure 1

Internally the wave carpet will be a networked grid, of interconnected redundant independent power take off units. Let us look at each of these qualifications. Each node on the artist's impression is a self-contained power-take off mechanism. For the moment the type of power take off mechanism is not critical but all that matters for our design is that the loading on the device should be within our control so that the control of the interconnected mesh of devices can be optimized to satisfy our total system goals. Each of the power take off mechanisms is small in size compared to present day wave energy devices. Hence the wave carpet will contain an enormous number of such power units. They will all be internally connected electrically, in a serial parallel configuration enhancing the redundancy of the design. Thus the failure of some units will not appreciably affect the overall performance of the Wave Carpet. This conceptual design also reduces maintenance at sea which, as we have seen in current designs can be as high as 20%. This close spacing of power take off devices in the wave carpet though not the best hydrodynamic absorption design, is a better design from the view of maintenance and eventually overall life cycle cost. The redundant devices and the parallel connections necessitate only down grading the power plant rating in extreme cases of damage thereby avoiding maintenance costs. Closely moored array of devices are considered to be hydro-dynamically efficient but have maintenance problems in extreme weather due to entanglement of mooring lines.

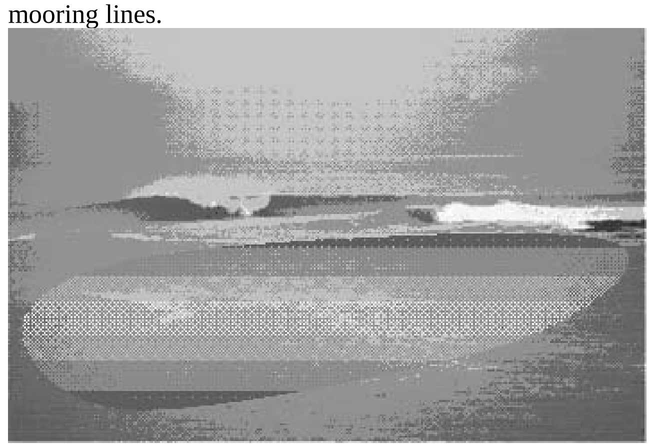

Figure 1: The "Wave Carpet"

The wave carpet design is better favored as it is a monolithic block without these mooring line problems. The biggest benefit of the wave carpet is at deployment where due to its monolithic structure once towed and deployed in position there is comparatively less work to start the plant. There are negligible inter-device positioning and connection problems. Also redeployment to another site is only a matter of towing using tugs.

Another advantage with this design is that in deep waters most devices are incapable of absorbing from multidirectional short crested seas, but because of the small size of the power take off units in the wave carpet in comparison to short waves this design will be better suited for such environments. The Wave Carpet does not have any preferred direction or orientation, as it will be designed to absorb from any direction. The waves traveling under the Carpet will get attenuated as it passes under. Each of the power take-off units act, as a damper slowly extracting whatever power it is capable of. The wave slowly but effectively though not necessarily efficiently will be eaten up by the Wave Carpet.

The specifics of the power take off mechanism are not so very critical to the benefits of the overall design. Some possible power take off mechanisms include piezoelectric materials or dielectric polymers for direct mechanical to electric conversion.
We present our ideas on how we could integrate the carpet with distributed small power take off devices. Our intention is that we could plug devices into the carpet as technology progresses. Figure 2 shows the general schematic of power conversion from Wave to Wire or Electricity.

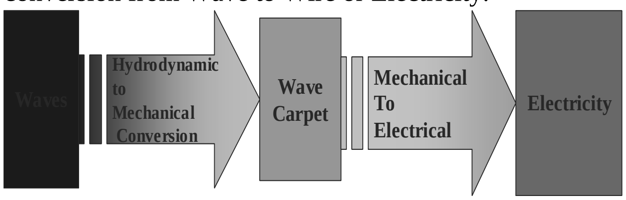

Figure 2: Wave to Wire System Chain

III. Modeling of Wave Carpet Dynamics

In this section we develop a coupled hydro-elastic time-domain solution for wave generation and carpet motion in a tank. The numerical tank concept for wave modeling has received considerable attention in the past few years[10][11]. We adopt the concept to the numerical tank simulation of transient motion of the Wave Carpet.

We formulate the problem in the Cartesian coordinate system (x,y,z) with 'x' the direction of wave propagation and along the free surface, 'y' along the width of the carpet and 'z' the vertical direction positive upwards. Let S2 be the surface of the tank lying under the "carpet" and S6 the remaining free surface on the top of the tank. Let 't' be the time dimension.

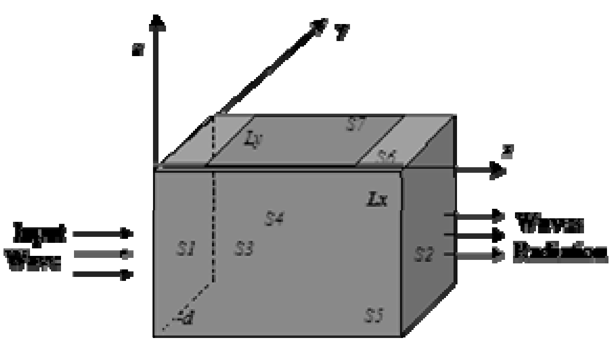

Figure 3.Scheme of tank (S1- boundary on which waves are generated, S2 - boundary subjected to radiation condition, S3, S4 (sides of the tank y=0 and y=Ly) subjected to non flow condition, S5 (tank's bottom) subjected to non-flow condition, S7 (carpet) subjected to equation (4), S6 (surface of the tank free from a the carpet) subjected to equation (3).

Assume that thickness of the carpet is ignored and $w(x,y,t)$ is the deflection of the carpet. In this case initial scheme Figure 3 can be presented as follows

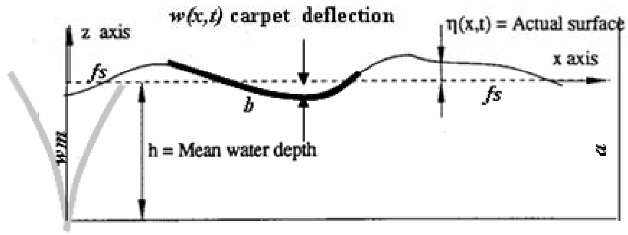

Figure 4. 2-D scheme of the model of numerical tank, *wm*- wave maker, b-beam as a model of a carpet, *fs*-free liquid surface, *a*-outlet boundary.

We assume an inviscid, incompressible and irrotational flow described by a velocity potential $\Phi(x,y,z,t)$ satisfying the Laplace equation in the domain.

$$\nabla^2\Phi(x,y,z,t) = 0 \quad in \quad 0 < x < L_x; 0 < y < L_y; -d < z < 0 \tag{1}$$

With the following boundary conditions:

$$\Phi_z\Big|_{z=-d} = \frac{\partial\Phi}{\partial z}\Bigg|_{z=-d} = 0 \quad and \quad \Phi_y\Big|_{y=0,Ly} = \frac{\partial\Phi}{\partial y}\Bigg|_{y=0,Ly} = 0 \tag{2}$$

First equation in (2) is a non-flow condition at the bottom $z = -d$. Second symmetry (isolation) condition in (2) is at the sidewalls of the tank $y = 0, y = Ly$.

Next from Euler equation follows Bernoulli

$$\Phi_t + \frac{1}{2}(grad\,\Phi)^2 + \frac{P}{\rho} + gz = 0 \tag{3}$$

Linearization of the Bernoulli equation.
Assume $z = \eta(x,t)$ is an equation for water surface,
Let : $(grad\,\Phi)^2 << \Phi_t$, and

$$\left|\frac{\partial^n \eta(x,y,t)}{\partial t^n}\right| << \left|\frac{\partial \eta(x,y,t)}{\partial t}\right|$$

$$(grad\,\Phi)^2 << \Phi_t$$

From this assumption it follows:

$$\begin{cases} \dfrac{\partial \Phi}{\partial t} + g\eta = 0 \\ \dfrac{\partial \Phi}{\partial z} = \dfrac{\partial \eta}{\partial t} \end{cases}$$

From equations above follow that

$$\frac{\partial \Phi}{\partial z} = -\frac{1}{g}\frac{\partial^2 \Phi}{\partial t^2}, \quad \text{on } S6 \qquad (4)$$

Pressure under the Wave Carpet is subjected an equation

$$P = -\rho \frac{\partial \Phi}{\partial t} - \rho g w \ \text{ on } S7 \qquad (5)$$

The carpet is modeled as an elastic beam characterized by its length Lx, width Ly, stiffness D, damping c, mass m and density ρ. We assume that the carpet behavior is subjected to Timoshenko-Euler equations[12]. For energy extraction we introduce a damping term.

$$m\frac{\partial^2 w}{\partial t^2} + c\frac{\partial \ w}{\partial t} + D\Delta^2 w + \rho g w + \rho \frac{\partial \Phi(x, y, 0)}{\partial t} = 0$$

$$\text{on } S7 \qquad (6)$$

The left side of (6) is the sum of the total force on the carpet, plus resistance by virtue of the energy extraction, plus shear forces and momentum appearing in the bending carpet, plus gravity force.

The conjugate condition between water surface and the carpet floating above is given by,

$$\Phi_z = -\frac{\partial w}{\partial t} \quad \text{on } S7 \qquad (7)$$

On the edge of the carpet is a free of stress condition,

$$\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial \tau^2} = 0, \quad and$$

$$\frac{\partial^3 w}{\partial n^3} + (2-\nu)\frac{\partial^3 w}{\partial \tau^2 \partial n} = 0 \qquad (8)$$

$$on \quad the \quad S7 - carpet \quad edges$$

Here $\tau \ \ and \ \ n$ are the tangent and normal to the carpet's boundary.

In addition we assume that on the left boundary at x = 0 a wave maker generates periodical waves. In general waves can be generated in two ways [11]

- By prescribing wave maker motion on the upstream boundary
- By specifying wave behavior on controlled boundary

First approach in general is ill posed and could lead to numerical instability of the solution of the problem in time varying domain. The second approach mathematically is well posed and allows simulation using time dependent boundary condition on the controlled boundary. Both approaches are mutually bounded and often can be reduced to each other, but we will use the latter. Hence the wave maker is modeled using properties of standard wave forms like a certain velocity distribution f0(y, z, t).

$$\left.\frac{\partial \Phi}{\partial x}\right|_{x=0} = f_0(y, z, t) \quad on \ S1 \qquad (9)$$

To satisfy a finite computational domain on the right side boundary x = Lx the flow satisfies the Sommerfeld radiation condition ensuring that the waves are outgoing[13],[14]. One approach for imposing Sommerfeld's radiation condition in a bounded reservoir was first developed by Orlanski [13]In the present work we use a modified Orlanski equation for wave radiation condition modeling on the boundary associated with infinity and is represented by,

$$\frac{\partial \Phi}{\partial x} = -\frac{1}{c}\frac{\partial \Phi}{\partial t} \ when \ \ x = L_x \quad on \ S1 \qquad (10)$$

This implies numerical computation of the phase velocity 'c': This representation guarantees a stable and unique solution of the corresponding equations, and also prevents the possible impact of the artificial boundary on the carpet/wave behavior.

Sides y = 0 and y = Ly are tank walls and we impose the non-flow condition in y direction on the S3 and S4

$$\left.\frac{\partial \Phi}{\partial y}\right|_{y=0, L_y} = 0 \qquad (11)$$

The Initial condition is,

$$\Phi(x, y, z, 0) = h(x, y, z) \qquad (10)$$

Equations (1) to (10) define the coupled hydro-elastic time-domain solution for wave generation and carpet motion in a tank As one can see initial boundary problem is not a classical one. Namely differential operator in domain is an elliptic of a second order however operator on the boundary contains second order derivatives with respect to time and fourth order derivative with respect to a spatial variable. To overcome this difficulties when we introduced four geometries and six coupled variable. And we used so called

The coupling variables concept in FEMLAB appeared to be extremely powerful in their ability to make the value and the exact Jacobian of an expression available non-locally.

We define coupling variables Φ and $\dfrac{\partial \Phi}{\partial z}$ in two steps. First define the source, that is Ω; then define the

destination, that is S_6 and S_7 within which you want to use the resulting variable for equations (7), and (6) correspondingly.
In this paper we have resolved the problem by applying so Called weak form modeling.
The distinguishing features of FEMLAB's weak form for us is that it makes possible: 1) to build models with PDEs (7,6, and 8) on boundaries S6, S7, and S2 correspondingly, an 2) build models with mixed space and time derivatives.
We resolved our initial boundary problem using advanced features of the FEMLAB 2.3 in two-dimensions with a certain restriction on the initial function h (x, 0, z). The two dependent variables w(x,t) and $\Phi(x,0,t)$ are coupled by conjugate condition (5) and through (4).

Next we present typical results from our numerical simulations. Our focus was to understand the response of the carpet for different damping values (c) and material properties like density (ρ) and stiffness (D). We define Response as Displacement of the free surface per unit Wave Amplitude.
Figure 5 is the Time Response of the free surface of the tank for c =1000, ρ = 0.01. The free surface is spatially represented as Distance / Carpet Length with the incoming wave (from 0 to 1), carpet (from 1 to 2) and outgoing wave (from 2 to 3). Hence the wavelength and carpet length are equal.
Multiple solutions are over plotted for different time steps. The time steps are chosen to coincide with the input wave period of 0.8 seconds. The first four time steps are essentially transient. Response at step number five and six are already close to each other. Time 5.7 is step number eight and is presented to illustrate the steady state behavior. At steady state, the response in front of the carpet at the wave maker (x =0) is almost unity whereas behind the carpet it dies down to a very low value. The carpet has absorbed the energy in the wave thereby reducing wave transmission.

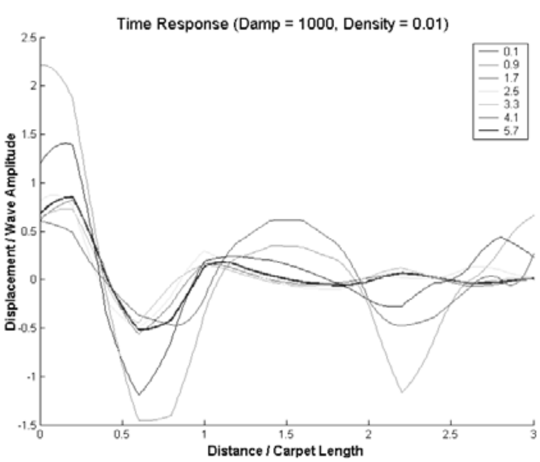

Figure 5: Time Response [c = 1000, ρ = 0.01]

Figure 6 shows the displacement at steady state for [c = 1000, ρ = 0.01]. The water depth is taken equal to the carpet length. The absorption of the carpet is clearer in this figure.

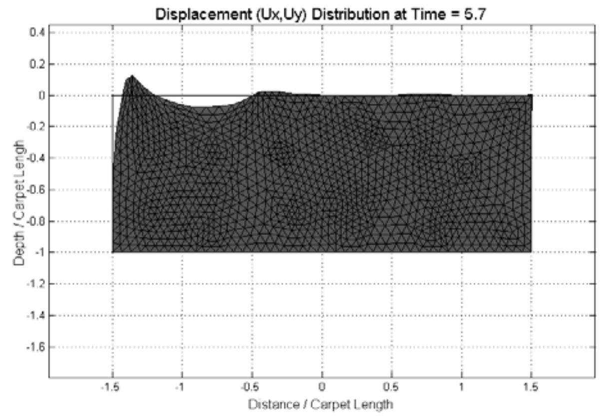

Figure 6: Steady State Displacement [c = 10^3, ρ = 0.01]

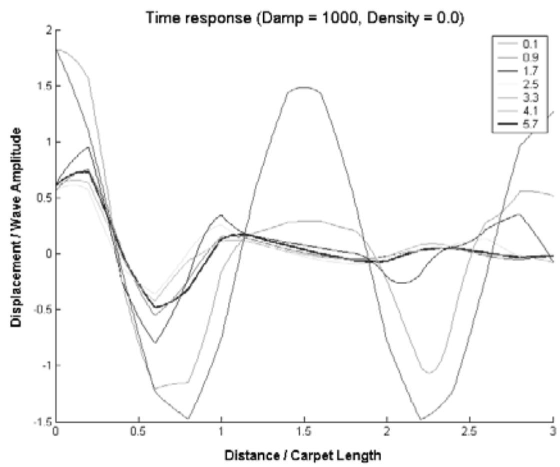

Figure 7: Time Response [c = 10^3, ρ = 0]

Figure 7 shows the time response as in Figure 6 but for ρ = 0. This change in density only affects the transient response; the steady state solution looks similar at time step 5.7.

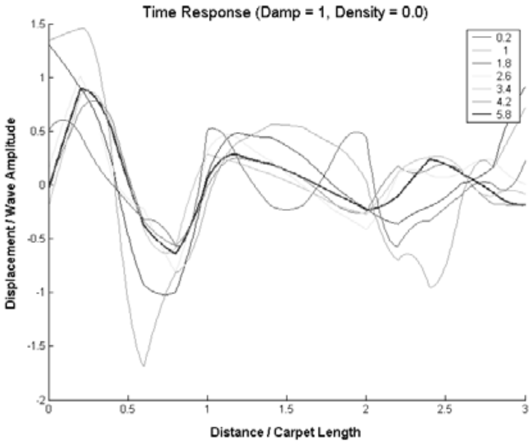

Figure 8: **Time Response [c = 1, ρ = 0]**

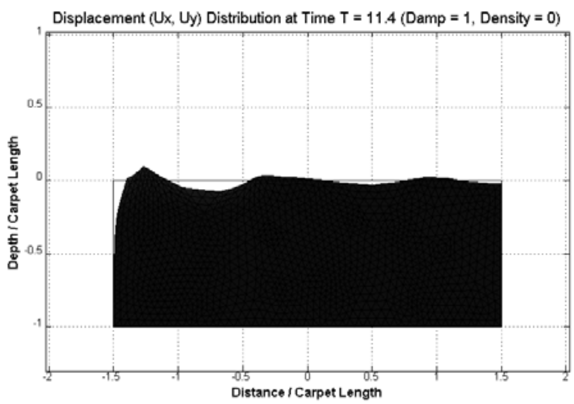

Figure 9: **Steady State Displacement [c = 1, ρ = 0]**

Figure 8 shows the response at damping c=1 and density zero. The low damping causes substantial portion of the wave to pass through the carpet as compared to Figure 5 and Figure 6 with high damping. Figure 9 shows the steady state solution for this low damping case and when compared to Figure 6 we can see the difference of carpet absorption.

IV. CONCLUSIONS

This paper presents a new deep-water wave energy device the Wave Carpet. Based on a review of past work we discuss the main issues that would be critical for such a deep-water rapidly redeployable design. We then conceptually introduce the wave carpet as a feasible solution.

We then propose a theory to model the carpet using the Numerical Wave Tank Concept and formulate the coupled hydro-elastic time-domain solution for wave generation and carpet motion in a tank. We then solve this problem using essential features and advances in FEMLAB in two dimensions and present typical results.

ACKNOWLEDGMENTS

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