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Disruptive Multi-Scale Mechanical Modelling of Micro-Structured Materials: Application to Car Seat Foam

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Abstract

Foam is a multiscale material composed of many millimetric cells. Altogether, the cells can easily be shaped into almost any object of the metric scale, provided that a mold is available to let the initial mixture expand appropriately.

While some mechanical properties such as foam density can analytically be calculated from the bulk material and the void ratio, others cannot. Even the Young modulus and Poisson ratio of an isotropic foam are out of reach for analytical analysis: in this work, it will be demonstrated that mixing laws fail at correctly predicting their values. Indeed, the structural properties depend strongly on the micro-structure of the foam.

In this work, periodic homogenization as described in Viry et al. [1] is applied to investigate the foam properties. The foam unit cell selected is based on Weaire and Phelan solids [2] that represent a periodical structure very close to foam structure. Indeed, the Weaire and Phelan pair of solids show two import properties: together, (a) they map the 3D space, and at the time of the study, (b) they are the periodical volumes verifying (a) that show the minimal surface to volume ratio. This latter property is mimicking accurately the energy minimization process involved in cell shape forming process.

Thanks to the periodic homogenization method, the Young modulus and Poisson ratio are computed and applied to a car seat under load. This allows for determining the seat shape under load. Moreover, periodic homogenization includes a relocation method, which is used to investigate the structural integrity of the cell wall and estimate the evolution of the foam properties over time. Therefore, such method is a powerful tool to study the replacement of traditional polymer-based composites and foams by environmentally respectful alternatives.

Keywords: foam, equivalent medium, multiscale approach, homogenization, elasticity, macroscopic and microscopic medium, unit cell, car seat.

1 Introduction

Foams are widely used in everyday life and industry as soft and lightweight materials. For certain applications, a specific formulation must be developed to obtain the desired properties and behavior: target porosity, user comfort, durability… *etc*. Numerical modelling could help to select relevant formulations while reducing the amount of experimental trials.

Using classical finite element techniques, foams can be very difficult to model directly due to the large number of tiny details required to describe the microstructure of an entire part. The use of specialized methods such as homogenization techniques is then required: Voigt and Reuss laws (also known as mixing laws) [3], or periodic homogenization [1].

The aim of this article is to propose a numerical workflow to help in the formulation of foams, by using homogenization methods. This workflow is illustrated by the following use-case: choosing the right foam density for a car seat.

2 Numerical Model

This work aims to optimize the formulation of a solid foam, that composes a car seat. This foam must have specific mechanical properties to maximize the user comfort, while ensuring its durability over time. Figure 1 provides an overview of the whole process.

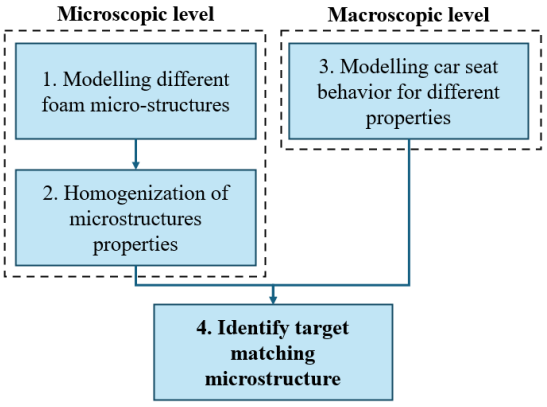

Figure 1. Overview of the numerical workflow.

Foam Micro-Structure Model

A solid foam is made of a stable cluster of gas bubbles separated by a solid film. Depending on the process and the chemical formulation, this cluster of gas bubbles can take many different shapes. Certain foams tend to minimize their surface/volume ratio. In this work, we use the Weaire and Phelan solids [2] that pave the space periodically as gas bubbles, and that show a very low surface to volume ratio (today, they even show the lowest ratio among all the solids periodically mapping the space). The boundaries of the solids are extruded to a specific thickness, thereby defining the solid film. The resulting periodic pattern for the solid phase is pictured in Figure 2. In this model, the film thickness is adjustable, and forms the parameter mimicking foam formulation variations.

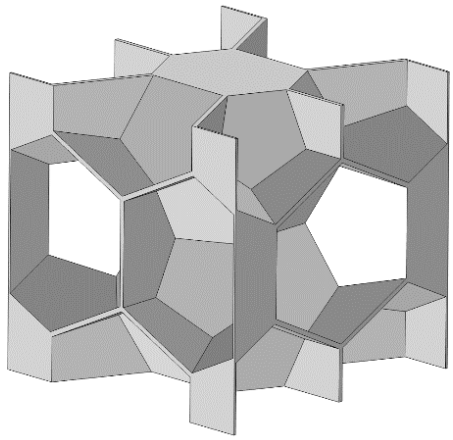

Figure 2. Periodic pattern of the solid foam micro-structure.

At the microscopic level, the solid has specific mechanical properties. Here, the following values are used: density $\rho = 1.45\ \mathrm{g/cm^3}$, Young modulus $E = 7.5$ MPa, and Poisson ratio $\nu = 0.499$, that are typical values for dense polyurethan. The holes contain air, and in this document, it is assumed that their mechanical effects are negligible. In the following, the periodic pattern of the solid film (Figure 2) and its mechanical properties form the *unit cell* of the microstructure.

Homogenization Methods

A part such as a car seat made of a foam may contain millions of repetitions of the unit cell, and its modelling without any simplifications is computationally out of reach: the use of homogenization methods is highly recommended. These methods allow to extract macroscopic mechanical properties (apparent density, Young modulus, Poisson ratio) by studying the microstructure. In the following, it is assumed that the solid film forming the foam remains in the elastic domain.

For a foam, the apparent density ρ is directly obtainable from the solid film density ρ_{solid} and the porosity of the material ε:

$$\rho = (1-\varepsilon)\,\rho_{solid} \tag{1}$$

Voigt and Reuss laws are simple laws to homogenize mechanical properties of composite materials. The Voigt law assumes that, under load, each phase of the unit cell is subjected to the same strain. For a two-phase material, the resulting macroscopic moduli, compressibility modulus k and shear modulus μ are [3]:

$$\begin{aligned} k_V &= k_1 + c_0 \cdot (k_0 - k_1), \\ \mu_V &= \mu_1 + c_0 \cdot (\mu_0 - \mu_1), \end{aligned} \tag{1}$$

where the subscript $\square_V$ stands for "Voigt", the index designates the phase, and c_0 is the volume fraction of phase 0. This is the average of the phase moduli weighted by the volume fraction. The Reuss law assumes that, under load, each phase is subjected to the same constraint, resulting in the following macroscopic moduli for a two-phase material [3]:

$$\begin{aligned} k_R &= \frac{k_0\,k_1}{k_0 + c_0(k_1 - k_0)}, \\ \mu_R &= \frac{\mu_0\,\mu_1}{\mu_0 + c_0(\mu_1 - \mu_0)}. \end{aligned} \tag{2}$$

where the subscript $\square_R$ stands for "Reuss". This is the harmonic average of the phase moduli weighted by the volume fraction. Applied to our unit cell, the first phase being the solid film (moduli k_{solid} and μ_{solid}), and the second phase being air ($E \approx 0$), the homogenized properties become:

$$\begin{aligned} k_V = (1-\varepsilon)\,k_{solid}, \quad \mu_V = (1-\varepsilon)\,\mu_{solid}, \\ k_R = 0, \quad \mu_R = 0, \end{aligned} \tag{3}$$

and can be expressed as a Young modulus and a Poisson ratio:

$$\begin{aligned} E_V = 3\,k_V(1-2\nu_V), \quad \nu_V = \frac{3\,k_V - 2\,\mu_V}{2(3\,k_V + \mu_V)}, \\ E_R = 0, \quad \nu_R = \text{undetermined}. \end{aligned} \tag{4}$$

Voigt and Reuss laws generally provide an interval containing the effective macroscopic properties of the microstructure. As seen in Eq. 3 and Eq. 4, this interval is expected to be rather large.

Periodic homogenization is a more sophisticated approach taking into account the geometrical specificities of the microstructure, rather than being based on very simplifying hypotheses. This method has multiple advantages: the resulting homogenized properties are proven mathematically as almost exact, the computational effort is rather low, and it does much more than homogenization. Indeed, the method also allows for a very accurate prediction of constraints inside the microstructure.

To set up the method, nine unitary displacement fields $\boldsymbol{\gamma}_{jk}$ must be solved within the unit cell:

$$\begin{aligned} -\boldsymbol{\nabla}\cdot\left[\mathbf{C}:\boldsymbol{\epsilon}_{jk}\right] &= \boldsymbol{\nabla}\cdot\left[\mathbf{C}:\boldsymbol{e}_{jk}\right], \\ \boldsymbol{\epsilon}_{jk} &= \tfrac{1}{2}\left[\boldsymbol{\nabla}\boldsymbol{\gamma}_{jk} + \left(\boldsymbol{\nabla}\boldsymbol{\gamma}_{jk}\right)^{\mathrm{T}}\right], \\ \boldsymbol{e}_{jk} &= \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & 1 & \vdots \\ 0 & \cdots & 0 \end{pmatrix}, 1 \text{ at } jk\text{-th component}, \\ j &= 1,2,3, \\ k &= j,\cdots,3, \end{aligned} \tag{5}$$

under $\boldsymbol{e}_{jk}$, the unitary strain, where $\mathbf{C}$ designates the elasticity tensor of the unit cell (function of space). These equations are closed with periodic boundary conditions and imposing mean value of $\boldsymbol{\gamma}_{jk}$ as zero. Six unitary displacements are in fact needed using the symmetry $\boldsymbol{\gamma}_{kj} = \boldsymbol{\gamma}_{jk}$. From these unitary displacements, the homogenized elasticity tensor $\mathbf{C}_h$ is obtained:

$$\mathbf{C}_{\boldsymbol{h}lmjk} = \tfrac{1}{V}\int_{\text{solid}} \mathbf{C}_{lmjk} + \left(\mathbf{C}:\boldsymbol{\epsilon}_{jk}\right)_{lm}\,\mathrm{d}\boldsymbol{x}. \tag{6}$$

with V, the volume of the unit cell and voids. This is an empirical adaptation of the method described in our former paper [1] for microstructures with holes. From this homogenized elasticity tensor, assuming isotropy, a Poisson ratio and a Young modulus can respectively be derived:

$$\begin{aligned} \nu_h &= \left(\frac{\mathbf{C}_{h_{1111}}}{\mathbf{C}_{h_{1122}}} + 1\right)^{-1}, \\ E_h &= \mathbf{C}_{h_{1111}}\frac{(1+\nu_h)(1-2\nu_h)}{1-\nu_h}. \end{aligned} \tag{7}$$

The method being empirically adapted to microstructures with holes, accuracy of the homogenized properties will be carefully checked.

Car Seat Mechanical Model

Figure 3. Car seat geometry.

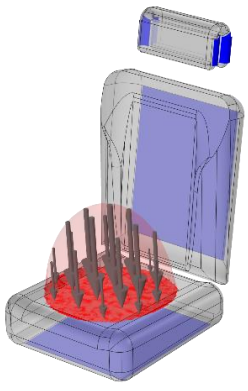

Figure 4. Car seat boundary conditions.

A schematic but representative car seat is selected for this work. It is composed of the seat itself, the backrest and the headrest. It is represented in Figure 3. The seat is approximately 50 cm wide.

The solid mechanics in the car seat foam is implemented using a linear elastic material assumption. The equation:

$$\boldsymbol{\nabla}\cdot\mathbf{S} = \mathbf{0} \tag{8}$$

with $\mathbf{S}$ the stress tensor, is resolved in the full domain represented in Figure 3. The actual frame is not considered in this work for the sake of simplicity.

The boundary condition of fixed constraint is applied at the bottom of the seat part, at the back of the backrest and on the side of the head rest, as seen in Figure 4 (blue surfaces). Such fixed constraint represents the frame action on the foam. In this work, the load is only applied on the seat. A circular contact Hertzian pressure distribution is applied on top of the seat, as seen in Figure 4 (red area, with grey arrow distribution). The diameter of the circular load is 40 cm, and the load integral is 838 N.

Foam Density Optimization Methodology

A parametric study is performed to quantify the effect of the apparent density of the foam on its mechanical properties. Then, multiple simulations of the car seat are performed with these properties, to evaluate the global behavior of the part, in function of the foam apparent density. Assuming the existence of criteria to optimize (global weight, user comfort, maximum constraints level… *etc*), a relevant foam formulation can be selected from these simulation results.

3 Results and Discussion

Comparison of Homogenization Methods

The homogenized properties obtained at a specific porosity (or equivalently a specific film thickness, or a specific density) using each homogenization method are given in Table 1. For the periodic homogenization method, the isotropy of the material has been verified by inspecting each component of the homogenized elasticity tensor $\mathbf{C}_h$.

Table 1. Homogenized properties obtained by each method with a porosity $\varepsilon = 0.913$.

	ρ (g/cm^3)	E (MPa)	ν (1)
Reuss law	0.127	0	N/A
Voigt law		0.655	0.499
Periodic homogenization		0.231	0.38

The Young modulus obtained using the periodic homogenization method falls inside the interval formed by the Voigt and Reuss laws. But this interval is extremely large, ranging from infinitely soft to relatively stiff. For this foam, there is a factor of 3 between the periodic homogenization Young modulus and the Voigt law one, which leads to very different behavior depending on the law selected. In order to select the most relevant law, a numerical experiment has been made to compare the behavior of a few repetitions of the unit cell (a tiny cube of foam of 3 mm size) under load, with its homogenized counterparts. The part is fixed at its bottom face and is subjected to a compression load homogeneously distributed over the top face. The results are pictured in Figure 5. The homogenized part issued from periodic homogenization almost perfectly reproduces the global behavior of the part in terms of axial compression and transversal dilatation, while the homogenized part issued from the Voigt law only captured the order of magnitudes. Thus, it is preconized to use periodic homogenization to accurately model the macroscopic behavior of foams.

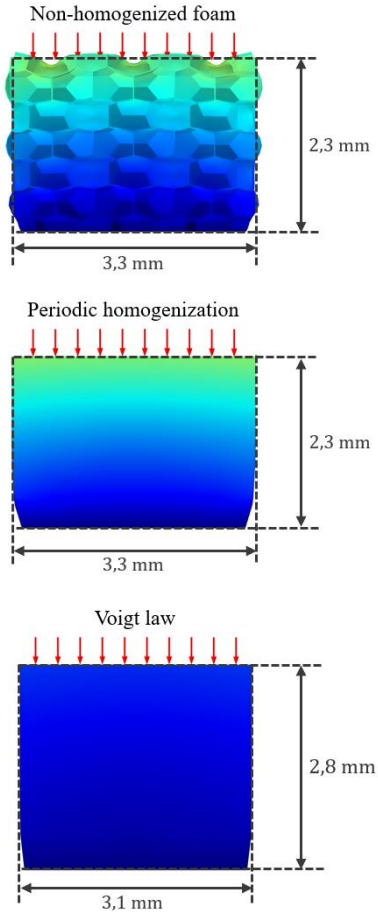

Figure 5. Comparison of the deformation of a non-homogenized foam and two homogenized foams modelled by the Voigt law and periodic homogenization, subjected to the same compression load.

Application to the Car Seat Case: Foam Optimization

The parametric study on the apparent density of the foam is performed using the periodic homogenization method. The results are given in Figure 6.

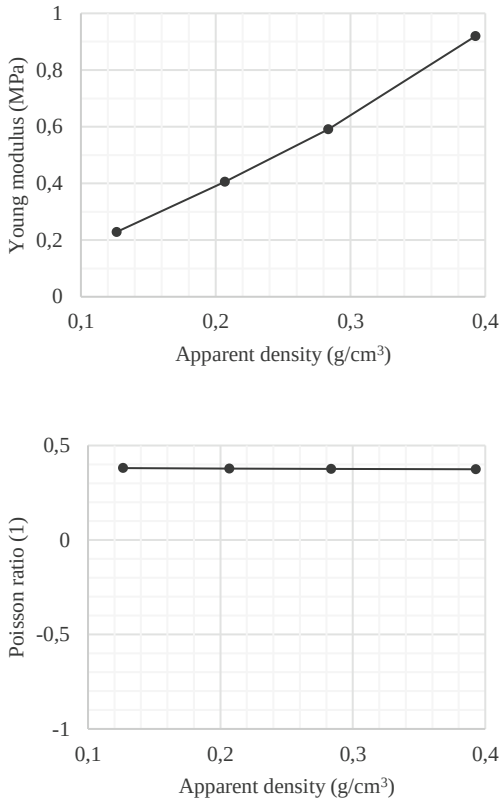

Figure 6. Foam mechanical properties obtained with the periodic homogenization method in function of its apparent density.

The Young modulus increases with the apparent density of the foam, which is not surprising, with values ranging from 0.1 to 1 MPa. The Poisson ratio does not depend on the apparent density of the foam, staying at value 0.38. The foam is more compressible than the bulk solid, probably due to its specific geometrical microstructure and maybe due to the simplification hypothesis regarding gas contribution.

As the foam is characterized, the car seat behavior can be simulated using mechanical properties that the foam could have. Multiple simulations of the car seat under load have been performed with a Young modulus having values ranging from 0.02 to 2 MPa, and a constant Poisson ratio of 0.38. Under the Hertzian load modelling a seated user, the maximum displacement in the seat is reported in Figure 7.

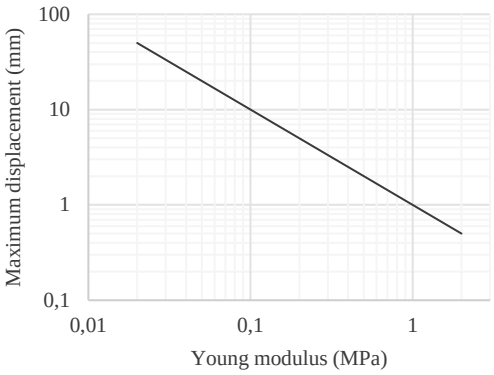

Figure 7. Maximum displacement in the foam under Hertzian load.

A target of arbitrarily 5 mm maximum displacement is selected in this load case. According to Figure 7, this means that the target Young modulus is 0.2 MPa. From Figure 6, this Young modulus is attainable with a foam having an apparent density of 0.1 g/cm^3. The full displacement field using this optimal foam is displayed in Figure 8.

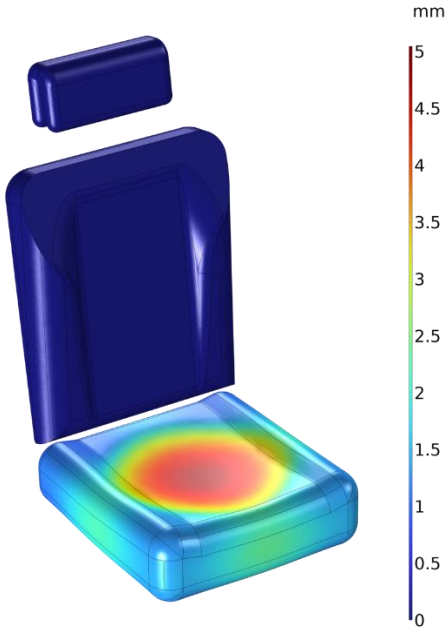

Figure 8. Displacement field of the car seat under the Hertzian load.

4 Conclusions

The aim of this work was to illustrate how homogenization methods can help industrials to formulate foams and to design parts made of foams.

A geometrical model of the microscopic structure of a foam was proposed using the Weaire and Phelan solids. It was then shown that the periodic homogenization method is very accurate in predicting the elastic properties of foams, while alternatives (Voigt and Reuss laws) are only able to predict the orders of magnitude. These numerical tools allowed for the study of the mechanical properties that the foam could have depending on its formulation. By simulating the mechanical deformations of a part made of foam, the optimal mechanical properties and thus its formulation were determined. Homogenization methods are then valuable numerical tools for evaluating and formulating foams. This method can alternatively be used for other materials defined by their microstructure: metallic foams, carbon fiber… *etc*.

Durability is another desired property that requires the ability to predict the constraints at the microscopic level and mechanical fatigue. This is part of our future work.

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