Coupling the transfer of light with the transfer of heat using a diffusion approximation

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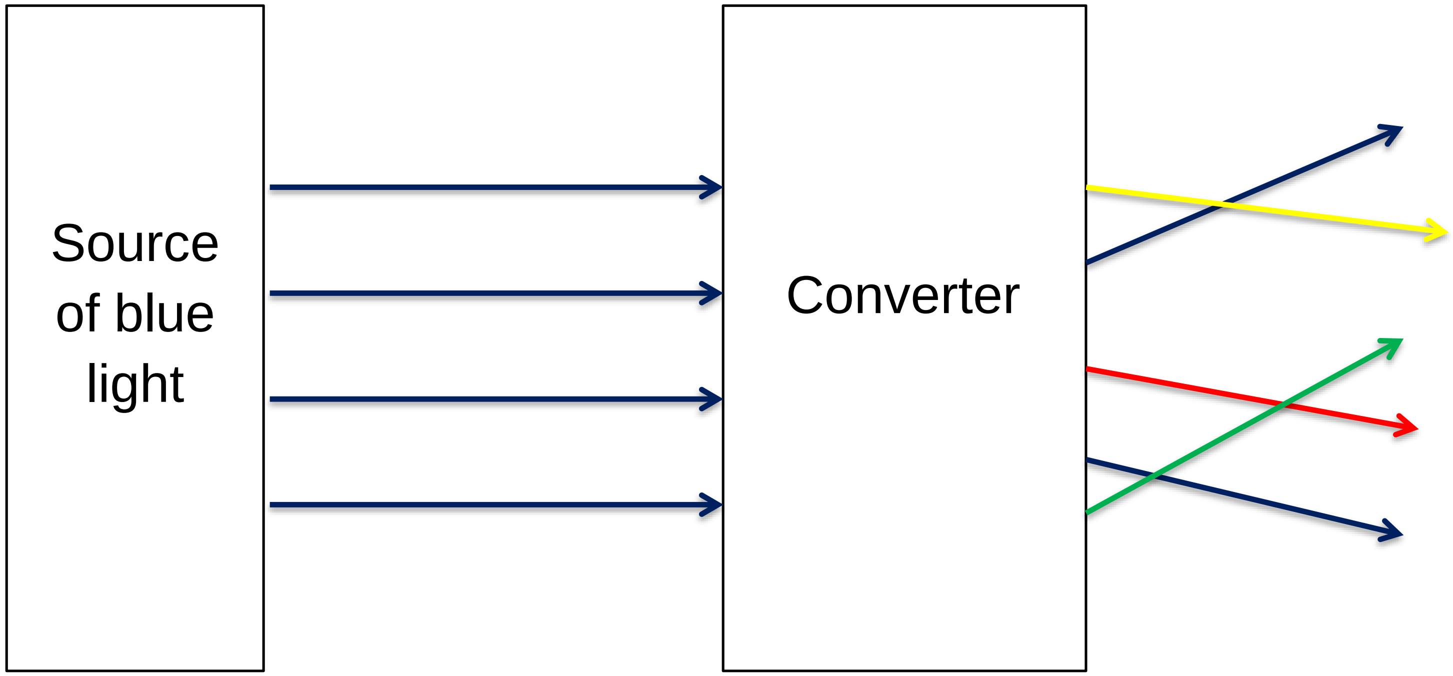
Outline

- **Converters in solid-state lighting**
- Radiance and why we should need to approximate it
- The diffusion approximation
- Coupling with heat transfer
- A simulation example in COMSOL

Converters in solid-sate lighting:
How to produce white light



What happens inside a converter?

- Absorption of impinging blue light
- Emission of absorbed light at longer wavelengths (green, yellow, red)
- Scattering of all light

Why model a converter?

We wish to predict the intensity and color of the light output, and how these metrics respond to changes in temperature.

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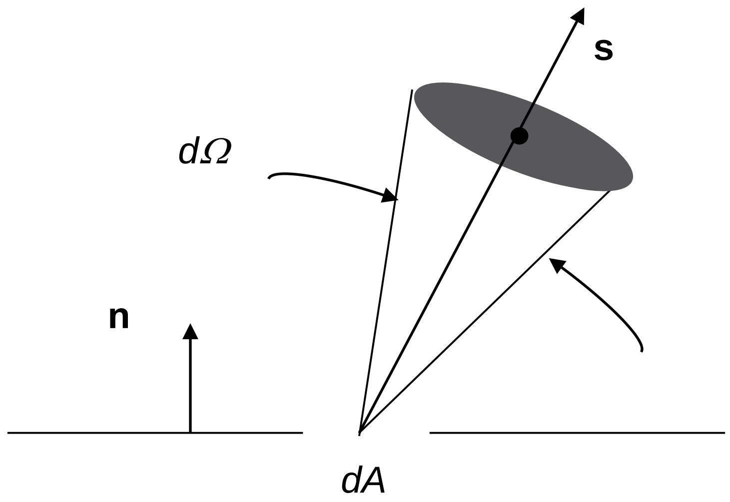
What is radiance?

For light of power *dP*, passing through a small aperture at location *r* and of area *dA* with unit normal *n*, in a direction that is within solid angle $d\Omega$ of the direction *s*, the radiance *L(r,s)* is a function of the location and direction, and it obeys the relation

$$dP(r,s) = L(r,s)\, n \cdot s \, dA \, d\Omega$$



$$\Delta P = L\, dA\, \mathbf{n} \cdot \mathbf{s}\, d\Omega$$

Radiance

It is the power of light per unit projected area and per unit solid angle.

Why should we approximate radiance instead of solving for it?

Radiance is governed by the known radiation transport equation.

$$L(r,s)$$

Radiance has two spatial arguments:
⇨ Large computing burden.

Approximation strategy:
Can we solve for functions of one spatial argument?

Radiation transport equation

Bulk volume absorptivity

Pore density

RTE for direction $\hat{\mathbf{s}}$

$$\hat{\mathbf{s}}\cdot\nabla L(\mathbf{r},\hat{\mathbf{s}}) = -\left(n_p\langle\sigma_{ext}\rangle+\alpha\right)L(\mathbf{r},\hat{\mathbf{s}}) + n_p\int d\Omega'\,L(\mathbf{r},\hat{\mathbf{s}}')\left\langle\frac{d\sigma_{sc}(\hat{\mathbf{s}},\hat{\mathbf{s}}')}{d\Omega'}\right\rangle + \frac{\varepsilon(\mathbf{r})}{4\pi}$$

Increase in radiance | Losses (scattering/absorption) | Increase from scattering | Isotropic emission

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Diffusion approximation*

1. Decompose radiation into two sorts: specular and diffuse. (Specular radiation propagates only in one direction; no *s* dependence.)

$$L(r,s) = L_s(r) + L_d(r,s)$$

2. For diffuse radiation, express the directional dependence as a power series in direction *s*:

$$L_d(r,s) = \frac{1}{4\pi}\underbrace{\Phi(r)}_{\text{Fluence}} + \frac{3}{4\pi}\underbrace{J(r)}_{\text{Current density}} \cdot s + [\text{Higher-order terms in } s\ldots]$$

3. Critical condition: Higher-order terms can be neglected in cases of high scattering:
   [Scattering length scale] << [Absorption and geometric length scales].
4. Now all unknowns (Specular radiance, fluence, current density) are only functions of *r*.

*Ishimaru, A., Wave Propagation and Scattering in Random Media, Volume 1 (Academic Press, Inc., San Diego), 175 – 188 (1978)
*A. Lenef, J. F. Kelso, Y. Zheng, and M. Tchoul, "Radiance limits of ceramic phosphors under high excitation fluxes," in SPIE Proceedings, Vol. 8841, Current Developments in Lens Design and Optical Engineering XIV, R. B. Johnson, V. N. Mahajan, and S. Thibault, Eds. (2013, paper 8841-6)

Diffusion approximation

Substitute approximate expressions into the radiation transport equation. We can then derive:

$$-D\nabla\Phi + J_0 = J \text{ [Diffusion relation]}$$

$$\frac{dL_s}{dz} + \left(\gamma_{abs} + \gamma_{scat}\right) L_s = 0 \text{ [Specular light]}$$

$$\nabla \cdot \left(-D\nabla\Phi + J_0\right) - \gamma_{abs}\Phi + \varepsilon_c + \varepsilon_s = 0; \text{ [Fluence]}$$

$J_0 =$ Specular current density

$D = \dfrac{1}{3\left(\gamma_{abs} + \gamma_{scat}\right)}$ "Diffusion" coefficient

$\gamma_{abs} =$ Absorption coefficient

$\gamma_{scat} =$ Scattering coefficient

$\varepsilon_c =$ Source of converted light from emission

$\varepsilon_s =$ Source of diffuse blue light from scattered specular light

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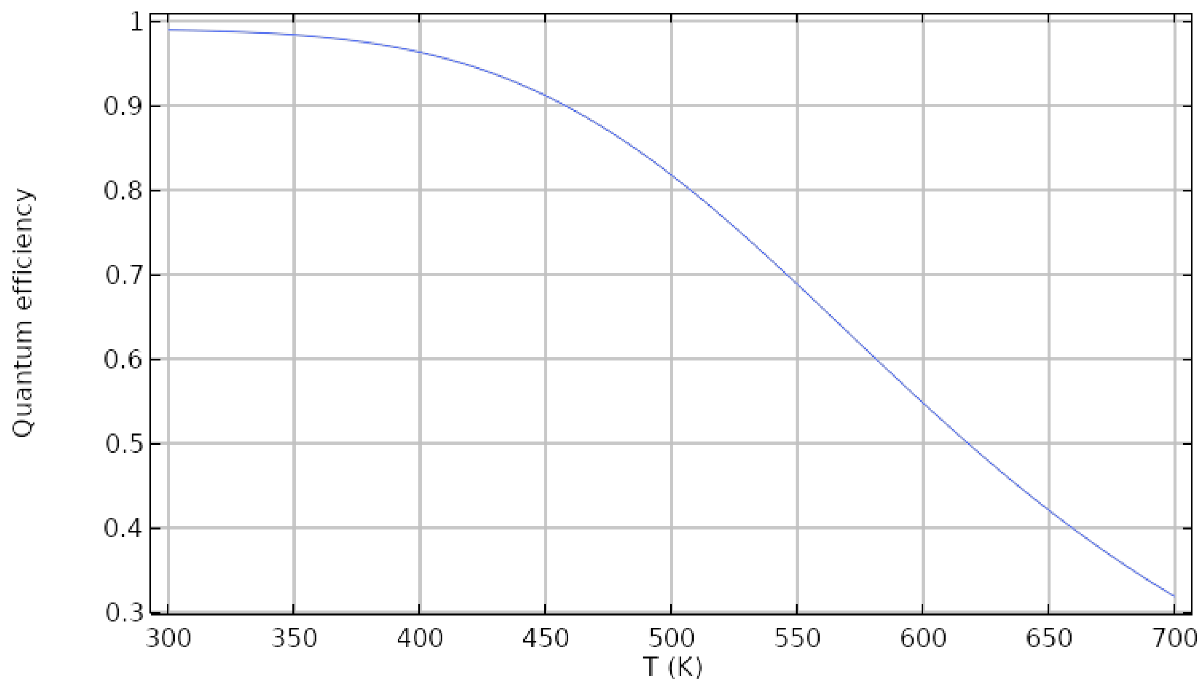
Why are we interested in thermal coupling?

When the converter gets too hot, performance degrades in two aspects:

- Absorption of blue light is reduced
- Quantum efficiency is reduced.

For an atom or particle that converts light, the quantum efficiency is defined as

$$QE = \frac{\#\ \text{Photons of converted light}}{\#\ \text{Photons of absorbed blue light}}$$



Common temperature dependence of quantum efficiency, used as in input in the model

This high-temperature reduction causes “rollover” in solid-state light sources.

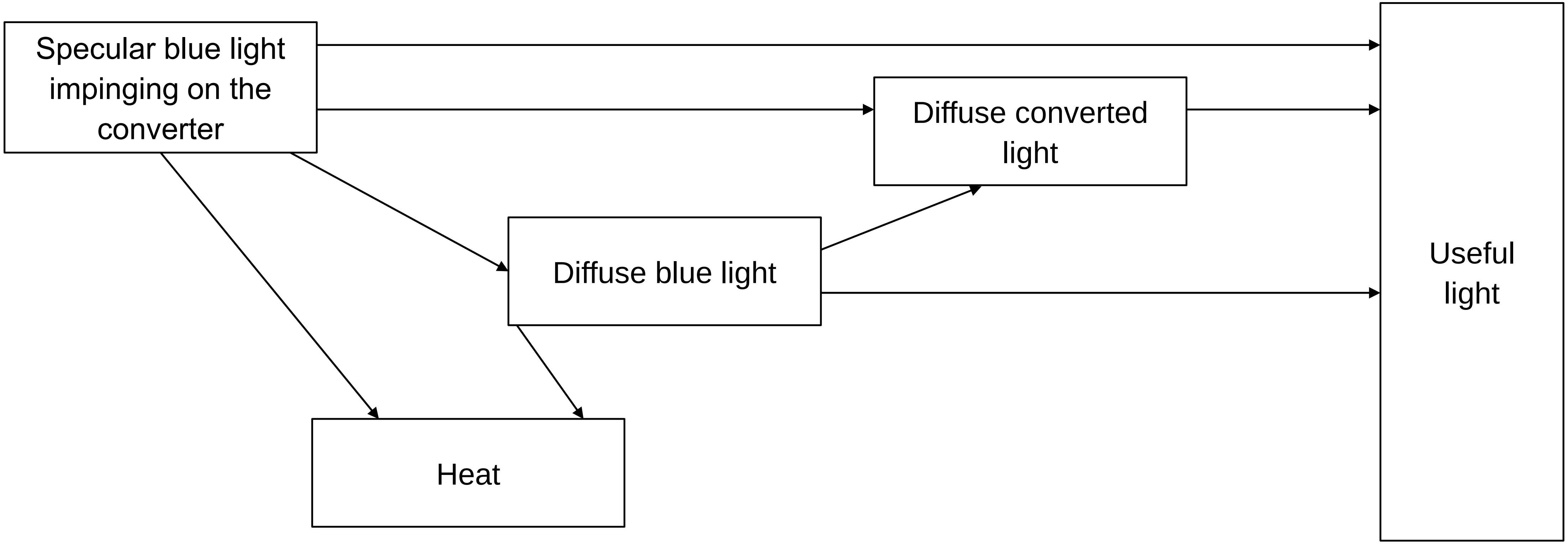
Two-way thermal coupling in the equations:

Heat equation: $$\nabla \cdot (\kappa \nabla T) + q = 0$$

Coupling from light to heat: Heat source q comes from absorbed light that is not emitted.

Coupling from heat to light: Temperature dependence appears in absorption γ_{abs} and in emission ε through the quantum efficiency.

Energy budget of a converter



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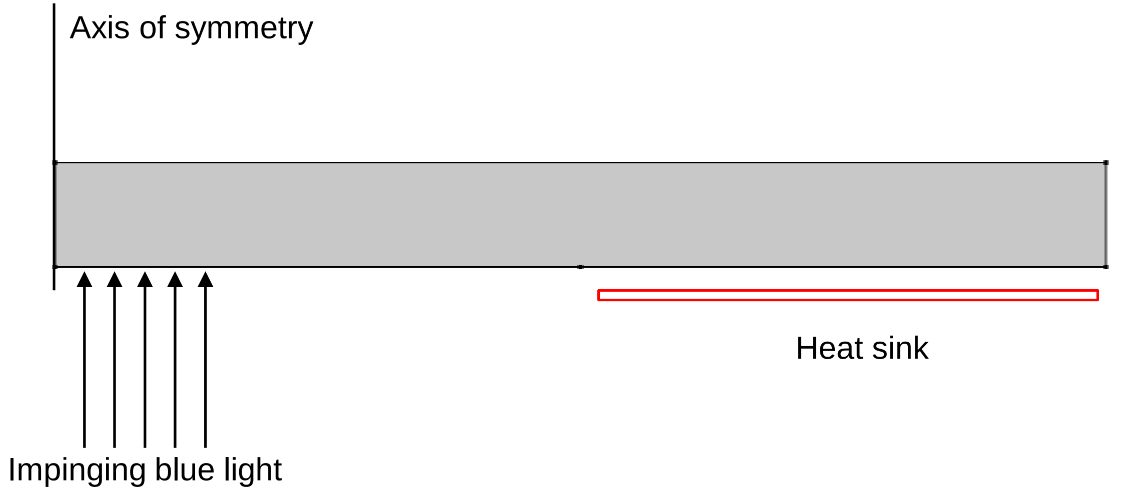
Problem formulation

- Idealized geometry that is not a real product, but exposes material to similar conditions.
- 1 to 6 W of blue specular light impinging on a disc of diameter 1 cm and thickness ½ mm.
- Axi-symmetric formulation.
- Neglect wavelength-dependence of material properties of converted light.
- 4 Coupled equations implemented in the PDE modes.
- Boundary condition on light: Fresnel transmission and reflection on exposed surfaces.
- Boundary condition on heat flow: Annular heat sink on the lower surface.



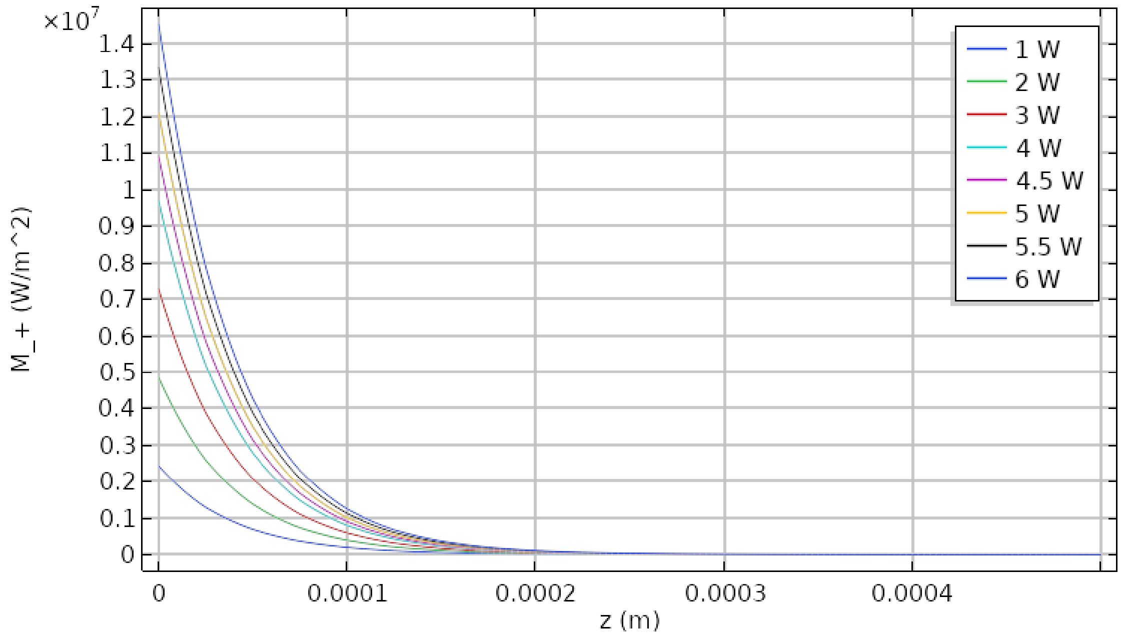
$$\frac{\partial M_+}{\partial z} + \gamma_{ext\_b} M_+ = 0 \qquad \text{Blue specular}$$

$$-\frac{\partial}{\partial r}\left(D_b r \frac{\partial \Phi_b}{\partial r}\right) - \frac{\partial}{\partial z}\left(D_b r \frac{\partial \Phi_b}{\partial z} - J_0 r\right) + \gamma_{abs\_b} r \Phi_b = r\varepsilon_b \qquad \text{Blue, diffuse}$$

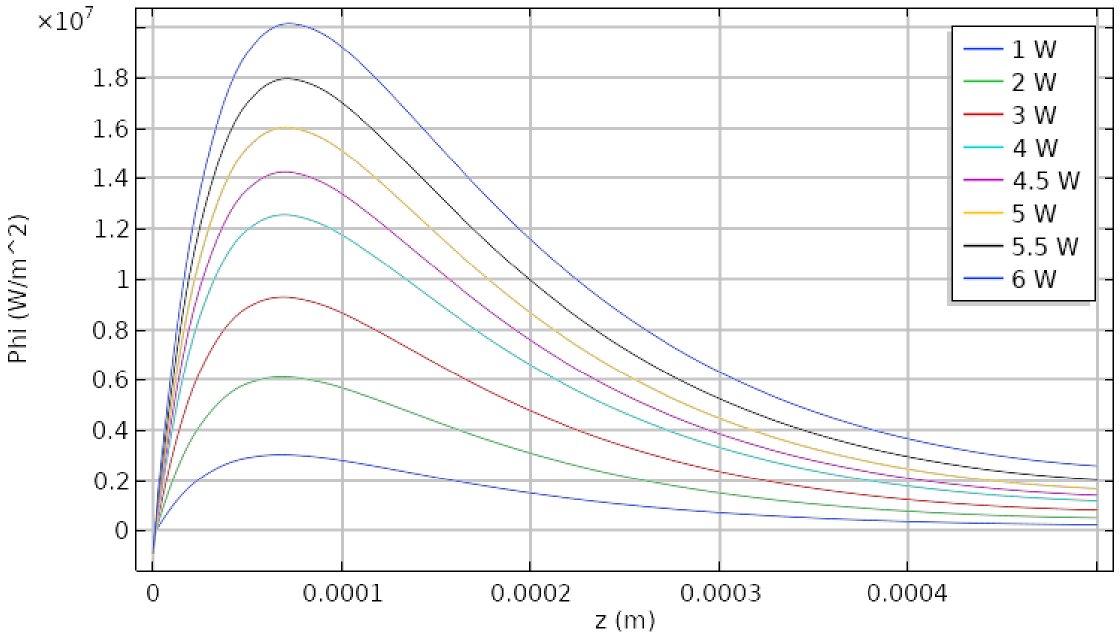
$$-\frac{\partial}{\partial r}\left(D_y r \frac{\partial \Phi_y}{\partial r}\right) - \frac{\partial}{\partial z}\left(D_y r \frac{\partial \Phi_y}{\partial z}\right) + \gamma_{abs\_y} r \Phi_y = r\varepsilon_y \qquad \text{Converted, diffuse}$$

$$-\frac{\partial}{\partial r}\left(\kappa_G r \frac{\partial T}{\partial r}\right) - \frac{\partial}{\partial z}\left(\kappa_G r \frac{\partial T}{\partial z}\right) = rQ \qquad \text{Heat}$$

Model output: Centerline light properties



Vertical specular radiance on the centerline



Blue Fluence on the centerline

- Physical interpretation of fluence: Integration of diffuse radiance over all directions

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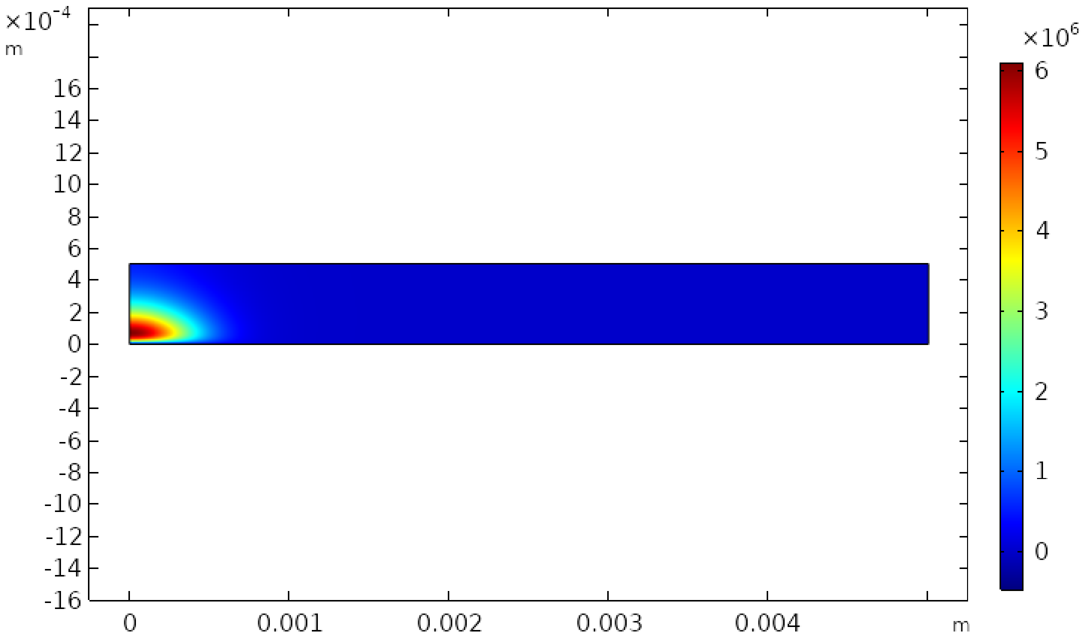
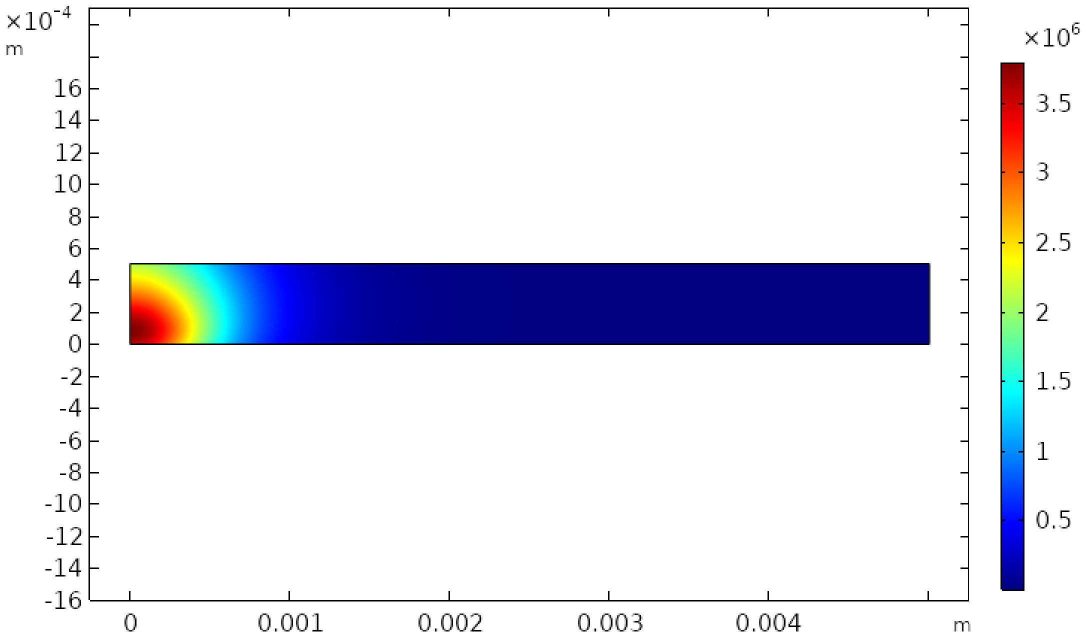
Model output



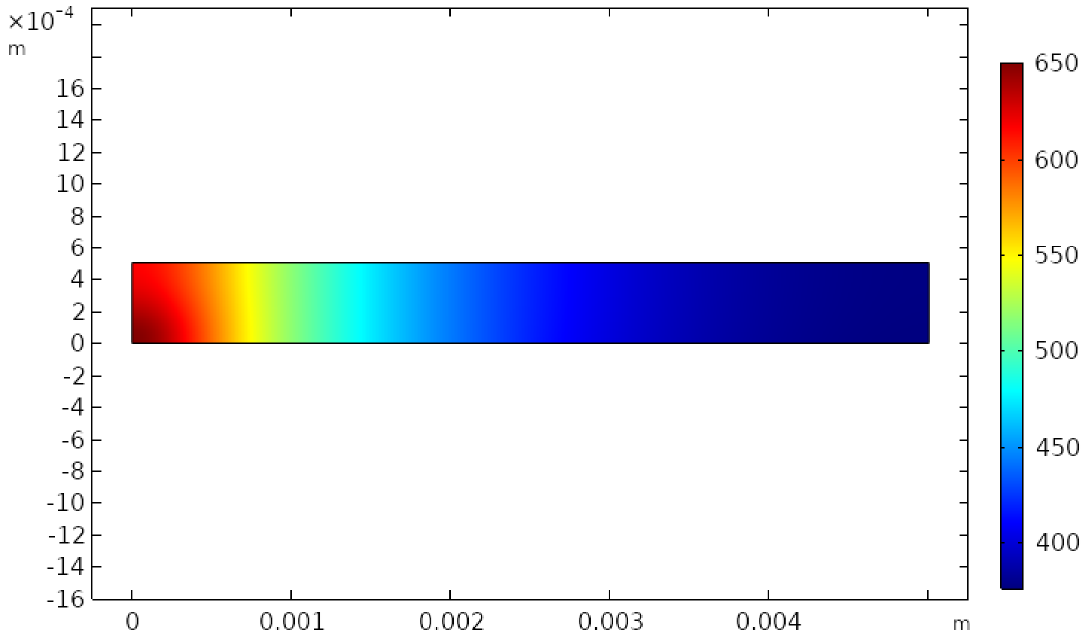
Blue fluence at 2 W



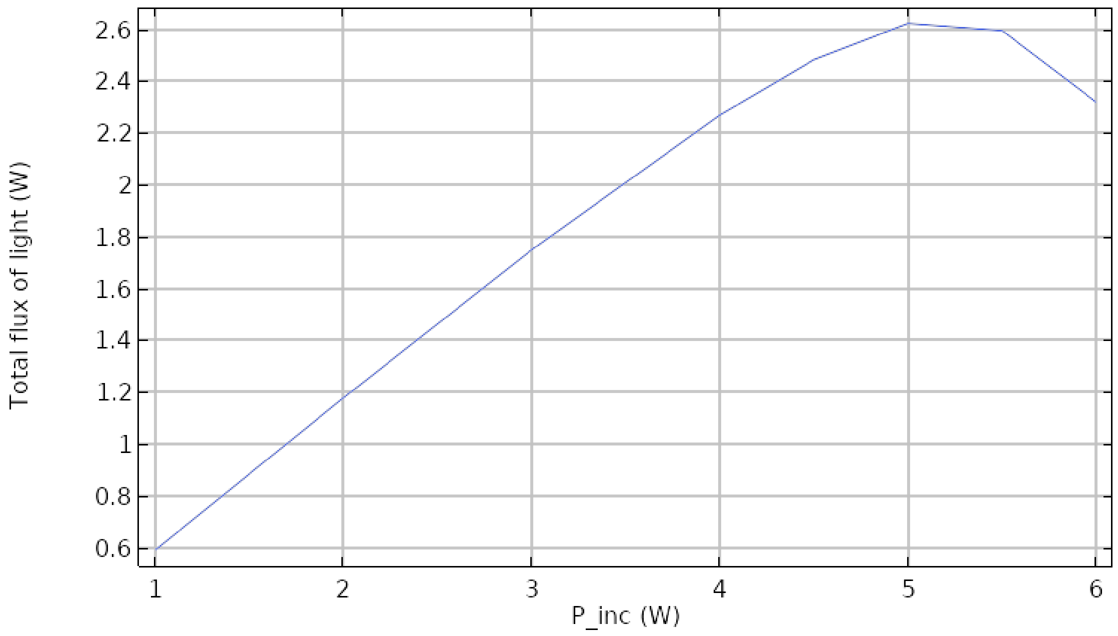
Converted fluence at 2 W

- Physical interpretation of fluence: Integration of diffuse radiance over all directions

Model output



Temperature (K) at 2 W



Total light output

Conclusion

- For converters with sufficiently-high scattering the diffusion approximation provides a plausible description of light propagation and its coupling to heat transfer.
- The diffusion equation can be solved very efficiently in COMSOL, with multiple coupled variables.
- Known effect of thermal rollover is reproduced with this method.

Thank you.